Lesson 19 Worksheet

October 13, 2017

For each problem:

a. find the largest open intervals on which $f(x)$ is concave up and concave down

b. find the inflection points (x, y)

c. find all relative extrema (use the second derivative test if possible)

1. $f(x) = (x^2 + 1)e^x$

2. $f(x) = \ln(\sqrt{x^2 + 3})$ (Hint: Simplify FIRST!)

3. $f(x) = \frac{1}{3}x^4 - \frac{1}{2}x^2 + 3$

4. $f(x) = x^4 + 4x^3 + 6x^2$

5. $y = \frac{1}{20}x^5 + \frac{1}{4}x^4 - \frac{2}{3}x^4 - 6x^2$ (don't do part (c))

Answers:

1. a. concave up on $(-\infty, -3) \cup (-1, \infty)$; concave down on $(-3, -1)$

   b. inflection points at $(-3, \frac{10}{e^3}), (-1, \frac{2}{e})$

   c. no relative extrema (2nd derivative test fails; 1st derivative test shows that $f(x)$ is always increasing)

2. a. concave up on $(-\sqrt{3}, \sqrt{3})$; concave down on $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

   b. inflection points at $(-\sqrt{3}, \ln \sqrt{6}), (\sqrt{3}, \ln \sqrt{6})$

   c. relative minimum at $(0, \ln \sqrt{3})$

3. a. concave up on $(-\infty, -1/2) \cup (1/2, \infty)$; concave down on $(-1/2, 1/2)$

   b. inflection points at $(-\frac{1}{2}, \frac{139}{48}), (\frac{1}{2}, \frac{139}{48})$

   c. relative minimum at $(-\frac{\sqrt{3}}{2}, \frac{45}{16}), (\frac{\sqrt{3}}{2}, \frac{45}{16})$; relative maximum at $(0, 3)$

4. a. concave up on $(-\infty, \infty)$

   b. no inflection points

   c. relative minimum at $(0, 0)$

5. a. concave up on $(-3, -2) \cup (2, \infty)$; concave down on $(-\infty, -3) \cup (-2, 2)$

   b. inflection points at $(-3, -\frac{279}{10}), (-2, -\frac{244}{15}), (2, -\frac{356}{15})$